

Electric potential & potential energy



Disclaimer : This PPT is intended only as a quick revision of formulas, definitions, theorems and theory concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT. Take active participation in your regular classes where you can enjoy learning physics through discussions, interactions, simulations, live demonstrations, derivations and problem solving

Electric potential

- ❑ Electric potential at a point is defined as the work done to bring a unit positive charge from infinity to that point.

$$V = \frac{W}{q}$$

- ❑ It is a scalar quantity
- ❑ SI unit is volt (V)

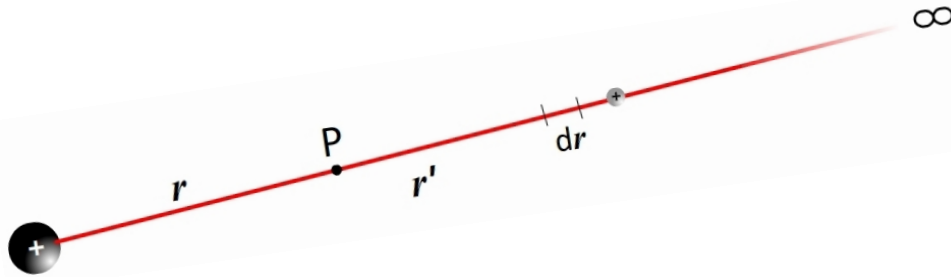


In practical applications potential difference is a relevant quantity (rather than just potential). In all cases potential at a point implies the difference between potential at that point and another reference point (such as infinity at which the potential is assumed to be zero). In some cases even reference breaks down !!

Electric potential and capacitance

Electric potential due to a point charge

Consider a charge Q (called source charge) fixed at a point in space. Let a small charge q (called test charge) be brought from infinity to a point P which is at a distance of r from the source charge.



At an instant let q be located at a distance r' from Q . Force acting on it at this point is

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r'^2} \hat{r}$$

Work done to displace q through dr is

$$dW = \mathbf{F} \cdot d\mathbf{r} = -\frac{1}{4\pi\epsilon_0} \frac{Qq}{r'^2} dr$$

Work done in displacing q from infinity to point P is given by

$$W = \int_{\infty}^r -\frac{1}{4\pi\epsilon_0} \frac{Qq}{r'^2} dr$$

$$W = -\frac{Qq}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{r'^2} dr$$

$$W = -\frac{Qq}{4\pi\epsilon_0} \left[-\frac{1}{r'} \right]_{\infty}^r$$

$$W = \frac{Qq}{4\pi\epsilon_0} \left[\frac{1}{r} \right]$$

Electric potential due to a point charge

Potential at a point is defined as the work done per unit charge therefore

$$V = \frac{W}{q}$$

$$V = \frac{Qq}{4\pi\epsilon_0} \left[\frac{1}{r} \right] \frac{1}{q}$$

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

- ☐ This is potential due to a point charge only
- ☐ For a negative charge potential will be negative

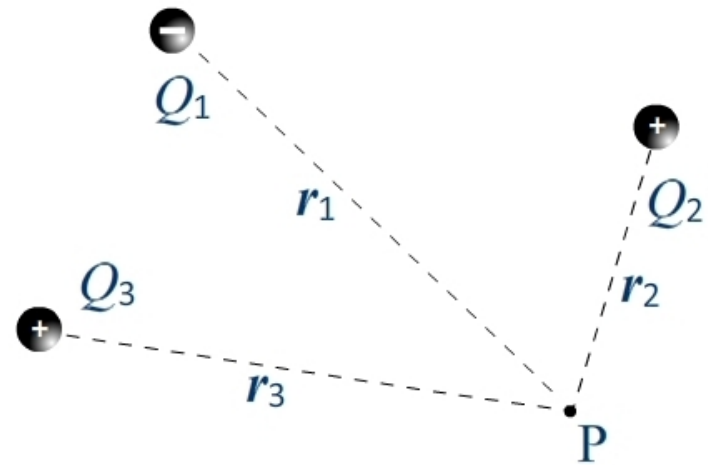
Electric potential and capacitance

Electric potential due to a system of discrete charges

Principle of superposition (for electric potential)

Electric potential at any point due to a collection of charges is the sum of potential due to each charge individually, at that point.

$$V = V_1 + V_2 + V_3 + \dots$$



Electric potential and capacitance

Equipotential surface

It is an imaginary surface on which potential is same at any point.

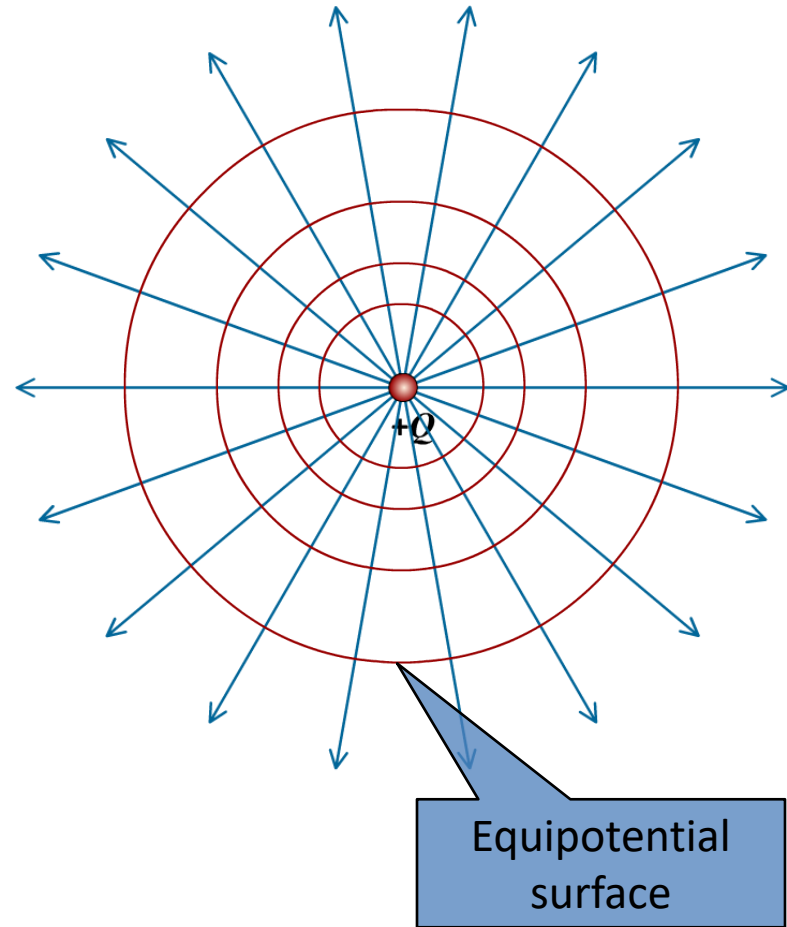
- Electric field at any point is always normal to the equipotential surface at that point.
- Work done to move a charge from one point to another on an equipotential surface is zero

Example :

For a single point charge, potential at a distance is given by the relation

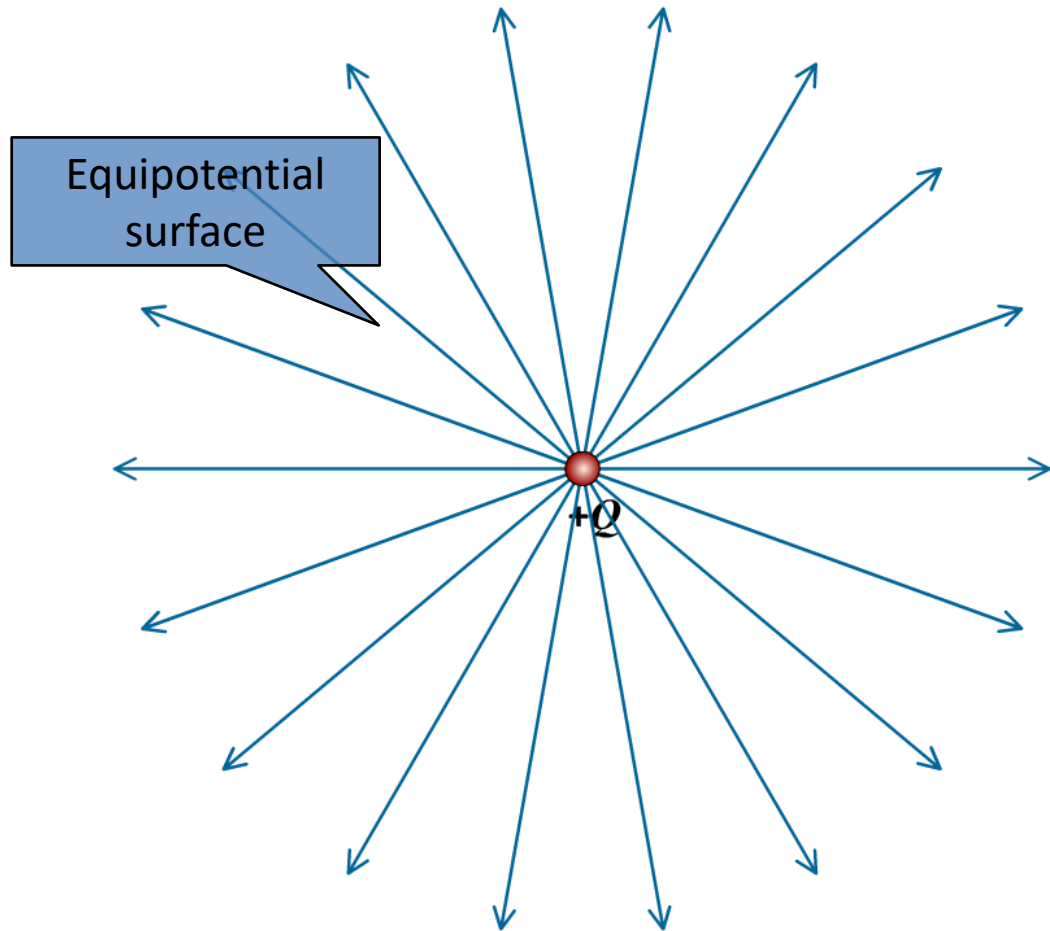
$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Potential is same at all points which are equidistant from the point charge. Therefore equipotential surfaces are a set of spheres concentric to the given charge.



Electric potential and capacitance

Equipotential surface (examples)



Electric potential and capacitance

Potential energy (electrostatic)



To bring the first charge

$$W = 0$$

To bring the second charge in the presence of the first charge

$$W_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0} \left[\frac{1}{r_{12}} \right]$$

To bring the third charge in the presence of the first and second charge

$$W = \frac{Q_2 Q_3}{4\pi\epsilon_0} \left[\frac{1}{r_{23}} \right] + \frac{Q_1 Q_3}{4\pi\epsilon_0} \left[\frac{1}{r_{13}} \right]$$

Total work done in assembling the charges is

$$W = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}} \right]$$

This work done, on the system, is stored as electrostatic potential energy therefore

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}} \right]$$

Note : Electrostatic force is conservative in nature , therefore $W_{\text{estat}} = - \Delta PE$

Relation between electric field intensity and electric potential

Electric field intensity at any point is the negative of gradient of scalar electric potential at that point.

$$\mathbf{E} = -\nabla V$$

$$\mathbf{E} = -\text{grad } V$$

$$\mathbf{E} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V$$

For a one dimensional situation, partial differentiation may be replaced with a regular differential

$$\mathbf{E} = -\frac{dV}{dx}$$

An often used (but erroneous) relation is

$$\mathbf{E} = \frac{V}{d}$$

Potential at a point (or potential difference between two points) can be calculated from $\mathbf{E}$ in the region using the relation

$$V = -\int \mathbf{E} \cdot d\mathbf{S}$$

Electric potential and capacitance

Some properties of conductors (electrostatic point of view)

- ❑ Electrostatic field inside a conductor is zero (defining property of a conductor)

If the field was not zero then the free electrons would move around and this redistribution would occur until the existing field (if at all) becomes zero.

- ❑ The interior of a conductor can have no excess charge in the static situation

Using Gauss law and the above argument, if E is zero then flux associated with any infinitesimal closed surface is zero. This implies that the net charge enclosed is zero.

- ❑ Electric field at any point on the surface on a charged conductor is always normal the plane of the conductor at that point.

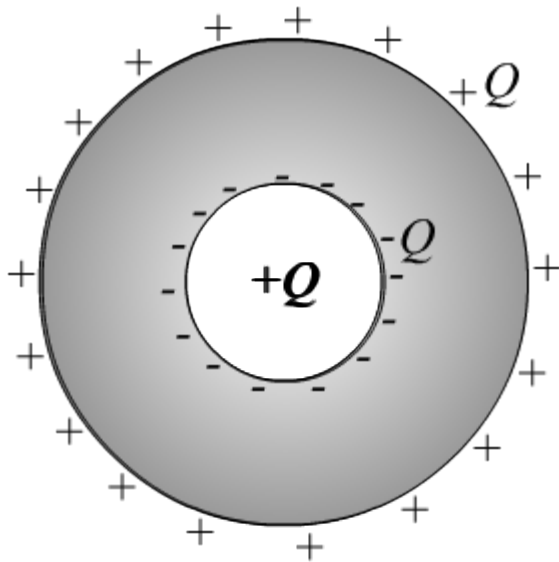
This is because if E was not normal to the surface then its tangential component would again lead to redistribution of charges until the tangential component of field (if at all) becomes zero.

- ❑ Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface (because E is zero)

- Conductors contain charges that are free to move (also called free charges)
- In solid conductors these are electrons and in electrolytes these are +ve and -ve ions

Electric potential and capacitance

Some properties of conductors (electrostatic point of view)



Initially the sphere is uncharged

A charge placed at the centre induces equal amount of charges of opposite sign on the inner surface (induction)

Accumulation of charges on the inner surface leads to equal amount of charges (of opposite sign again !) on the outer surface of the spherical enclosure.

Surface charge density on the inner surface

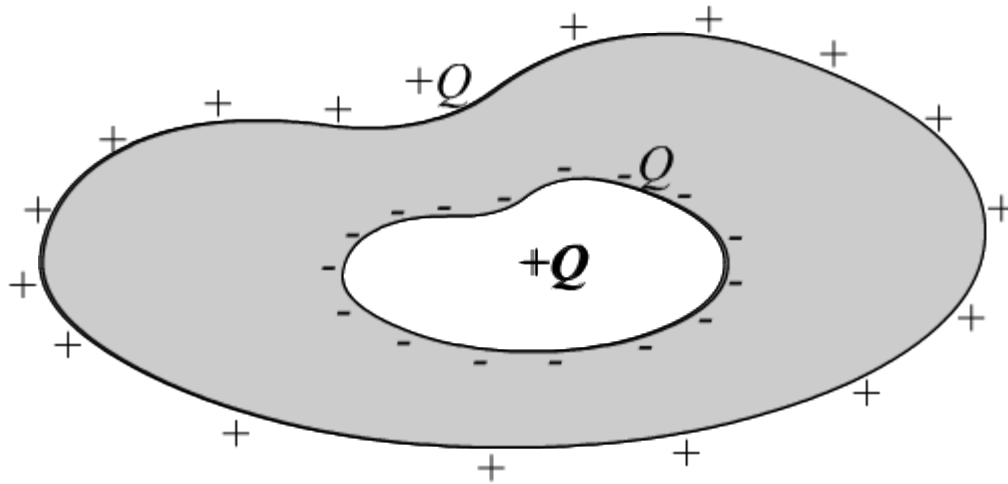
$$\sigma_1 = \frac{-Q}{4\pi r_1^2}$$

Surface charge density on the outer surface

$$\sigma_2 = \frac{Q}{4\pi r_2^2}$$

Electric potential and capacitance

Some properties of conductors (electrostatic point of view)



A charge placed at the centre induces charges of opposite sign on the inner surface

Accumulation of charges on the inner surface leads to equal amount of charges (of opposite sign) on the outer surface of the enclosure.

Surface charge density varies from region to region on both the inner and the outer surfaces. This surface charge density depends on the nature of the surface and the position of the charge placed in the cavity.

Electric field at the surface of a charged conductor

Consider the surface of a charged conductor. Let σ be the uniform surface charge density.

Consider a small Gaussian pill (cylindrical in structure)

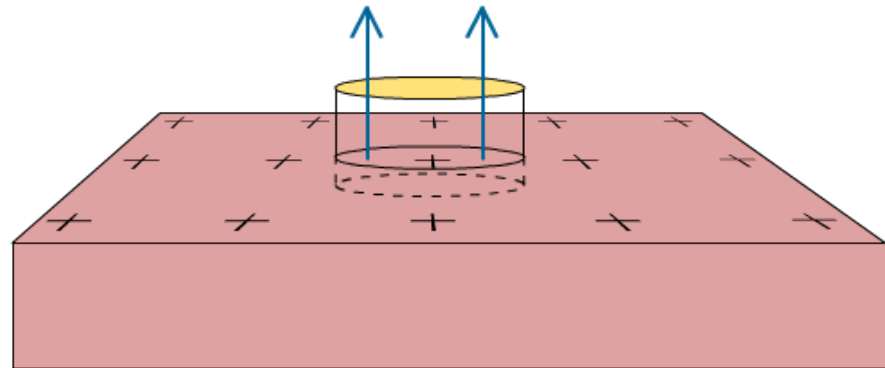
$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\int_{S_1} E \, dS \cos(\theta) = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E \int_{S_1} dS = \frac{\sigma A}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$



Electrostatic shielding

Electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed.

This is valid in all circumstances such as (a) the conductor is charged (b) induced charges appear on the conductor (c) the conductor is placed in a region of electric field.